> *Studying mathematics one rule at a time is like studying a language by first memorizing the vocabulary and the detailed linguistic rules, then building phrases and sentences, and only afterwards learning to read, write, and converse.*

Name and section: ______________________________

1. Consider $\mathbb{P}_3$ equipped with the inner product

$$\langle f(x), g(x) \rangle = \int_{-1}^{1} f(x)g(x)\,dx. \tag{1}$$

For example, $\langle x, x^3 \rangle = \int_{-1}^{1} x^4\,dx = \frac{2}{5}$. Apply Gram-Schmidt to the basis $\{1, x, x^2, x^3\}$ to find a basis of $\mathbb{P}_3$ which is orthogonal with respect to this inner product.

2. Label the following statements as true or false. (You do not need to justify your answers.)
   (a) (1 point) The projection of a vector b onto a subspace $\operatorname{span}(v_1, v_2)$ is $\frac{b \cdot v_1}{v_1 \cdot v_1} v_1 + \frac{b \cdot v_2}{v_2 \cdot v_2} v_2$.
   (b) (1 point) Every system $Ax = b$ has a unique least-squares solution.
   (c) (1 point) If A is an invertible $n \times n$ matrix and $v, w \in \mathbb{R}^n$, then $\langle v, w \rangle = (Av) \cdot (Aw)$ is an inner product on $\mathbb{R}^n$.
   (d) (1 point) The eigenvalues of the matrix $A = \begin{bmatrix} 2 & \pi & 17 \\ \pi & -100 & 42 \\ 17 & 42 & \sqrt{7} \end{bmatrix}$ are all real.
   (e) (1 point) If A is a matrix, then $A^T A$ is a symmetric matrix.